Vhodni signali

Trapezni impulz

$$U(\omega) = U_0 T_2 \frac{\sin\frac{\omega T_1}{2}}{\frac{\omega T_1}{2}} \frac{\sin\frac{\omega T_2}{2}}{\frac{\omega T_2}{2}} e^{-\frac{j\omega T}{2}}$$

T je trajanje impulza, T_1 je čas vzpona, T_2 je čas do začetka padanja

Beli šum

$$\phi_{vv}(\tau) = \Phi_0 \delta(\tau) \qquad \Phi_{vv}(\omega) = \Phi_0$$

Barvni šum

$$G_n(s) = \frac{1}{1+sT_g} \qquad T_g = \frac{1}{\omega_g}$$

$$\Phi_{nn}(\omega) = \left|G_n(j\omega)\right|^2 \Phi_0 = \frac{\Phi_0}{1+\left(\frac{\omega}{\omega_g}\right)^2} \qquad \phi_{nn}(\tau) = \frac{\Phi_0 e^{-|\tau|\omega_g}}{2}\omega_g$$

Zvezni naključni binarni signal

Povprečje menjav predznaka v časovni enoti:

$$P(n) = \frac{(\mu \Delta t)^n}{n!} e^{-\mu \Delta t}$$

Avtokorelacijska funkcija ima za $a^2 = \frac{\Phi_0}{2}\omega_g$ in $\mu = \frac{\omega_g}{2}$ enak potek kot širokopasovni šum.

Diskretni naključni binarni signal

$$\phi_{uu}(\tau) = \begin{cases} a^2\left(1-\frac{|\tau|}{\lambda}\right) & |\tau| < \lambda \\ 0 & |\tau| \geq \lambda \end{cases}$$

$$\Phi_{uu}(\omega) = a^2 \lambda \left(\frac{\sin\frac{\omega\lambda}{2}}{\frac{\omega\lambda}{2}}\right)^2$$

Psevdonaključni binarni signal

$$\phi_{nn}^d(\tau_i) = \begin{cases} a^2 & \tau_i = 0 \\ -\frac{a^2}{N} & 0 < \tau_i < N \end{cases}$$

$$\phi_{nn}^d(\tau_i \pm iN) = \phi_{nn}^d(\tau_i) \quad i = 0,1,2,\ldots$$

$$\Phi_{nn}^d(m) = \begin{cases} \frac{a^2}{N} & m = 0 \\ a^2(1+\frac{1}{N}) & 0 < m < N \end{cases}$$

Fourierova analiza

Moten prehodni pojav:

$$E\left\{\left|\Delta G_n(j\omega)\right|^2\right\} = \sigma_{G_{abs}}^2(\omega) = \frac{\Phi_{nn}(\omega)}{\Phi_{uu}(\omega)} \doteq \frac{\Phi_{nn}(\omega) T_A}{\left|U(\omega)\right|^2 m}$$

$$\sigma_{G_{abs}}(\omega) = \frac{\sqrt{\Phi_{nn}(\omega) T_A}}{\left|U(\omega)\right|\sqrt{m}} \qquad \sigma_{G_{rel}}(\omega) = \frac{\sqrt{\Phi_{nn}(\omega) T_A}}{\left|G(j\omega)\right|\left|U(\omega)\right|\sqrt{m}} \qquad \sigma_{G_{rel}}(\omega) = \frac{\left|G_n(j\omega)\right|\sqrt{\Phi_0 T_A}}{\left|G(j\omega)\right|\left|U(\omega)\right|\sqrt{m}}$$

Napačna ocena ustaljenega stanja (beli šum):

$$\sigma_{G_{abs}}(\omega) = \frac{\sqrt{\Phi_0}}{\omega\left|U(\omega)\right|\sqrt{T_B m}}$$

Analiza frekvenčnega odziva

Vrednotenje frekvenčnega odziva z vzorčevalnikom z vzorčenjem v dveh časovnih trenutkih

$$t_1 = nT_p = n\frac{2\pi}{\omega_0} \qquad t_2 = nT_p + \frac{T_p}{4} = (n+\tfrac{1}{4})\frac{2\pi}{\omega_0} \qquad n = 1,2,3,\ldots$$

$$y(t_2) = U_0\left|G(j\omega_0)\right|\cos\varphi = U_0 \operatorname{Re}\left\{G(j\omega_0)\right\}$$

$$y(t_1) = U_0\left|G(j\omega_0)\right|\sin\varphi = U_0 \operatorname{Im}\left\{G(j\omega_0)\right\}$$

Metoda ortogonalne korelacije

$$u(t) = U_0 \sin(\omega_0 t) \qquad y(t) = U_0\left|G(j\omega_0)\right|\sin\left(\omega_0 t + \varphi(\omega_0)\right)$$

$$\hat{\phi}_{uu}(\tau) = \frac{U_0^2}{2}\cos(\omega_0\tau) \quad \hat{\phi}_{uy}(\tau) = \frac{U_0^2}{2}\left|G(j\omega_0)\right|\cos\left(\omega_0\tau + \varphi(\omega_0)\right)$$

$$\hat{\phi}_{uy}(0) = \frac{U_0^2}{2}\left|G(j\omega_0)\right|\cos\left(\varphi(\omega_0)\right) = \frac{U_0^2}{2}\operatorname{Re}\left\{G(j\omega_0)\right\}$$

$$\hat{\phi}_{uy}(-\tfrac{\pi}{2\omega_0}) = \frac{U_0^2}{2}\left|G(j\omega_0)\right|\sin\left(\varphi(\omega_0)\right) = \frac{U_0^2}{2}\operatorname{Im}\left\{G(j\omega_0)\right\}$$

Vpliv motenj pri ortogonalni korelaciji

1. Visokofrekvenčni kvazistacionarni šum

    (a) beli šum $n(t) = v(t)$:

$$\sigma^2_{G_{v_{abs}}} = \frac{4\Phi_0}{U_0^2 n T_p}$$

(b) širokopasovni šum, dobljen iz belega šuma s filtriranjem preko filtra s pasovno širino $\omega_g = \frac{1}{T_g}$ (aproksimacija pri majhnih T_g in dolgih merilnih časih):

$$\sigma^2_{G_{n_{abs}}} \doteq \frac{4\Phi_{nn}(\omega_0)}{U_0^2 n T_p}$$

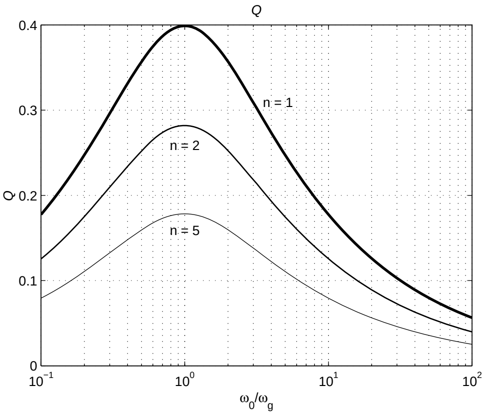
$$\sigma_{G_{n_{abs}}}(\omega) = E\left\{\left|\Delta G(j\omega_0)\right|\right\} \doteq \frac{\sqrt{2\Phi_0\omega_g}}{U_0} Q \qquad \sigma_{G_{n_{rel}}}(\omega) = \frac{E\left\{\left|\Delta G(j\omega_0)\right|\right\}}{\left|G(j\omega_0)\right|} \doteq \frac{\sqrt{2\Phi_0\omega_g}}{\left|G(j\omega\ )\right| U_0} Q \qquad Q = \frac{\sqrt{\frac{\omega_0}{\omega_g}}}{\sqrt{\pi\left(1+\left(\frac{\omega_0}{\omega_g}\right)^2\right)}} \frac{1}{\sqrt{n}}$$

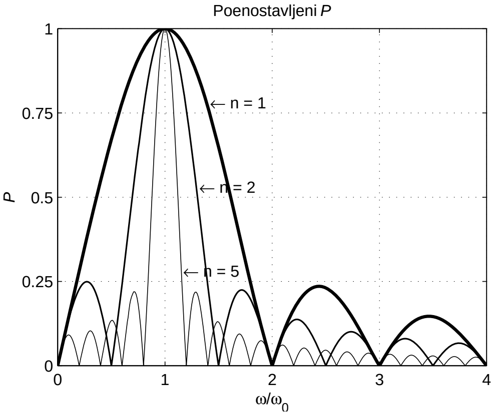
2. Harmonična (sin/cos) motnja

$$n(t) = N_0 \cos(\omega t)$$

$$\sigma_{G_{s_{abs}}} = \left|\Delta G(j\omega_0)\right| \doteq \frac{N_0}{U_0} P \quad \sigma_{G_{s_{rel}}} = \frac{\left|\Delta G(j\omega_0)\right|}{\left|G(j\omega_0)\right|} \doteq \frac{N_0}{U_0\left|G(j\omega_0)\right|} P$$

$$P_{poenostavljeni} = \frac{\sqrt{2}\,\frac{\omega}{\omega_0}\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}{\left|1-\left(\frac{\omega}{\omega_0}\right)^2\right|} \frac{\left|\sin(\pi\frac{n\omega}{\omega_0})\right|}{\pi\frac{n\omega}{\omega_0}} \quad \omega \neq \omega_0$$





3. Nizkofrekvenčne motnje – lezenje

$$n(t) = d(t)$$

$$\Delta G(j\omega_0) = \frac{2j}{U_0 n T_p} D(\omega_0) \quad \sigma^2_{G_d}(\omega_0) = \frac{4}{U_0^2 n^2 T_p^2}\left|D(\omega_0)\right|^2$$

Linearno lezenje

$$n(t) = D_0 t \quad \sigma^2_{G_d}(\omega_0) = \frac{4D_0^2}{U_0^2\omega_0^2}$$

Korelacijska analiza

<u>Zvezni prostor:</u>

$$\phi_{uy}(\tau) = \int_0^\infty g(t)\phi_{uu}(\tau - t)dt$$

$$u(t) = \text{beli šum} \quad g(\tau) = \frac{1}{\Phi_{u0}}\phi_{uy}(\tau)$$

$$\sigma_g^2 = \text{var}\left[g(\tau)\right] = \frac{1}{T}\left[\int_0^\infty g^2(t)dt + \frac{\sigma_n^2}{\Phi_{u0}}\right]$$

kjer je $\sigma_n^2 = \phi_{nn}(0)$ varianca šuma, ki moti izhodni signal

<u>Diskretni prostor:</u>

$$\phi_{uy}(\tau) = \sum_{k=0}^{\infty} g(k)\phi_{uu}(\tau - k)$$

$$\mathbf{\Phi}_{uy} = \mathbf{\Phi}_{uu}\mathbf{g}$$

$$\sigma_g^2(\tau) = \text{var}\left[g(\tau)\right] = \frac{1}{N}\left[\sum_{k=0}^{\infty} g^2(k) + \frac{\sigma_n^2}{\Phi_{u0}}\right]$$

kjer je $\sigma_n^2 = \phi_{nn}(0)$ varianca šuma, ki moti izhodni signal

Metoda najmanjših kvadratov

$$\mathbf{y} = \mathbf{\Psi}\boldsymbol{\theta} + \mathbf{v}$$

$$\hat{\mathbf{y}} = \mathbf{\Psi}\hat{\boldsymbol{\theta}}$$

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{\Psi}^T\mathbf{\Psi}\right)^{-1}\mathbf{\Psi}^T\mathbf{y}$$

$$\text{cov}\left[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right] = \sigma_e^2\left(\mathbf{\Psi}^T\mathbf{\Psi}\right)^{-1}$$

Tabela Laplaceove in z-transformacije

$x_z(t)$	$x(k)=x_z(kT)$	$\mathcal{L}\{x_z(t)\}$	$\mathcal{Z}\{x(k)\}$
	$\delta(kT)$		1
1	1	$\frac{1}{s}$	$\frac{z}{z-1}$
t	kT	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
t^2	k^2T^2	$\frac{2}{s^3}$	$\frac{T^2z(z+1)}{(z-1)^3}$
t^3	k^3T^3	$\frac{6}{s^4}$	$\frac{T^3z(z^2+4z+1)}{(z-1)^4}$
t^n	k^nT^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}\frac{\partial^n}{\partial a^n}\frac{z}{z-e^{aT}}$
e^{-at}	e^{-akT}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
te^{-at}	kTe^{-akT}	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
t^2e^{-at}	$k^2T^2e^{-akT}$	$\frac{2}{(s+a)^3}$	$\frac{T^2ze^{-aT}(z+e^{-aT})}{(z-e^{-aT})^3}$
t^ne^{at}	$k^nT^ne^{akT}$	$\frac{n!}{(s-a)^{n+1}}$	$\frac{\partial^n}{\partial a^n}\frac{z}{z-e^{aT}}$
$1-e^{-at}$	$1-e^{-akT}$	$\frac{a}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{a^2}{s^2(s+a)}$	$\frac{(aT-1+e^{-aT})z^2+(1-aTe^{-aT}-e^{-aT})z}{(z-1)^2(z-e^{-aT})}$
$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{b-a}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\sin\omega_0 t$	$\sin\omega_0 kT$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\frac{z\sin\omega_0 T}{z^2-2z\cos\omega_0 T+1}$
$\cos\omega_0 t$	$\cos\omega_0 kT$	$\frac{s}{s^2+\omega_0^2}$	$\frac{z(z-\cos\omega_0 T)}{z^2-2z\cos\omega_0 T+1}$
$e^{-at}\sin\omega_0 t$	$e^{-akT}\sin\omega_0 kT$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\frac{ze^{-aT}\sin\omega_0 T}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$
$e^{-at}\cos\omega_0 t$	$e^{-akT}\cos\omega_0 kT$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\frac{z^2-ze^{-aT}\cos\omega_0 T}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$